Analysis of the Dispersion Characteristic of Slot Line with Thick Metal Coating

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***Abstract*—A theoretical method is presented for the analysis of the slot line employing the network analytical methods of electromagnetic fields and Galerkin's procedure. The propagation constants for the dominant and higher order mode as well as the effect of the metal-coating thickness on the propagation constant and the characteristic impedance are obtained. The numerical results are compared with other available data.**

I. Introduction

The DISPERSION characteristic of slot line has been investigated by several authors [1]–[5]. Most of these theories, however, have treated the propagation of the dominant mode and have neglected the effect of the metal-coating thickness. The propagation of the higher order modes in slot line has been only approximately investigated [5], and the effect of the metal-coating thickness of slot line has been analyzed only for the propagation constant of the dominant mode [3].

This paper presents a method of analysis of the slot line with metal-coating thickness greater than zero. This method is an extention of the treatment in [3] using the hybrid mode formulation, and hence, is capable of giving the propagation constant for the higher order mode as well as the effect of the metal-coating thickness on the propagation constant and the characteristic impedance. Our analysis employs the network analytical methods of electromagnetic fields [6] for the derivation of integral equations and Galerkin's procedure [7] for the numerical computation. The method itself is quite general and applies to a number of other structures, although the results are not presented here.

II. Formulation of Integral Equations

The cross section of slot line to be analyzed is shown in Fig. 1. It consists of a slot in a metal coating on a dielectric substrate. In the following formulation, it is assumed that the metal coating and the dielectric substrate are lossless.

First we express the transverse fields in the regions (1) $z>t$, (2) $t>z>0$, (3) $0>z>-h$, and (4) $-h>z$ by the following Fourier integral:

A) regions (1), (3), and (4): $-\infty<x<\infty$

$$\left.\begin{matrix} \boldsymbol{E}_t^{(i)} \\ \boldsymbol{H}_t^{(i)} \end{matrix}\right\} = \sum_{\ell=1}^{2} \int_{-\infty}^{\infty} e^{-j\beta_0 y} \left\{ \begin{matrix} V_\ell^{(i)}(\alpha;z) & \boldsymbol{f}_\ell^{(0)}(\alpha;x) \\ I_\ell^{(i)}(\alpha;z) & \boldsymbol{g}_\ell^{(0)}(\alpha;x) \end{matrix} \right\} d\alpha, \quad i=1,3,4 \tag{1}$$

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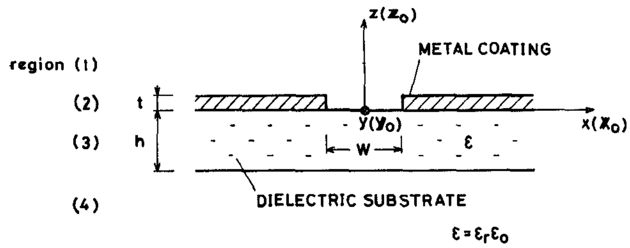

Fig. 1. Slot line. ϵ is the dielectric constant.

where

$$\boldsymbol{f}_1^{(0)} = \frac{j}{\sqrt{2\pi}\,K}(\boldsymbol{x}_0\alpha + \boldsymbol{y}_0\beta_0)e^{-j\alpha x}$$

$$\boldsymbol{f}_2^{(0)} = \frac{j}{\sqrt{2\pi}\,K}(\boldsymbol{x}_0\beta_0 - \boldsymbol{y}_0)e^{-j\alpha x}$$

$$\boldsymbol{g}_\ell^{(0)} = \boldsymbol{z}_0 \times \boldsymbol{f}_\ell^{(0)}\ (\ell=1,2), \qquad K=\sqrt{\alpha^2+\beta_0^2}\,. \tag{2}$$

B) region (2)

a) $|x|<W/2$

$$\left.\begin{matrix} \boldsymbol{E}_t^{(2)} \\ \boldsymbol{H}_t^{(2)} \end{matrix}\right\} = \sum_{\ell=1}^{2} \sum_{n=0}^{\infty} e^{-j\beta_0 y}\epsilon(n) \left\{ \begin{matrix} V_\ell^{(2)}(\alpha_n;z) & \boldsymbol{f}_\ell^{(c)}(\alpha_n;x) \\ I_\ell^{(2)}(\alpha_n;z) & \boldsymbol{g}_\ell^{(c)}(\alpha_n;x) \end{matrix} \right\} \tag{3}$$

where

$$\epsilon(n) = \begin{cases} 1/\sqrt{2}\,, & (n=0) \\ 1, & (n\neq 0) \end{cases}$$

$$\boldsymbol{f}_1^{(c)} = \frac{-1}{K_n}\sqrt{\frac{2}{W}}\ \{\boldsymbol{x}_0\alpha_n\cos(\alpha_n x) - \boldsymbol{y}_0 j\beta_0 \sin(\alpha_n x)\}$$

$$\boldsymbol{f}_2^{(c)} = \frac{1}{K_n}\sqrt{\frac{2}{W}}\ \{\boldsymbol{x}_0 j\beta_0\cos(\alpha_n x) - \boldsymbol{y}_0\alpha_n \sin(\alpha_n x)\}$$

$$\boldsymbol{g}_\ell^{(c)} = \boldsymbol{z}_0 \times \boldsymbol{f}_\ell^{(c)}, \qquad (\ell=1,2)$$

$$\alpha_n = \frac{2n\pi}{W}, \qquad K_n=\sqrt{\alpha_n^2+\beta_0^2}\,. \tag{4}$$

b) $|x|>W/2$

$$\boldsymbol{E}_t = \boldsymbol{H}_t = 0 \tag{5}$$

where β_0 is the propagation constant, $\boldsymbol{x}_0$, $\boldsymbol{y}_0$, and $\boldsymbol{z}_0$ are the x-, y-, and z-directed unit vectors, respectively, and $\ell=1$ and $\ell=2$ refer to the E waves ($H_z=0$) and the H waves ($E_z=0$), respectively. $V_\ell^{(i)}$ and $I_\ell^{(i)}$ are mode voltages and currents, and $\boldsymbol{f}_\ell^{(0)}$, $\boldsymbol{g}_\ell^{(0)}$, $\boldsymbol{f}_\ell^{(c)}$, and $\boldsymbol{g}_\ell^{(c)}$ are vector mode functions which satisfy boundary conditions at $x=\pm W/2(t>z>0)$ and $x=\pm\infty(z>t$ and $z<0)$ and have the following orthonormal properties:

$$\int_{-W/2}^{W/2} g_{\ell'}^{(c)*}(\alpha_{n'};x)\cdot z_0\times f_\ell^{(c)}(\alpha_n;x)\,dx=\delta_{\ell\ell'}\delta_{nn'}$$

$$\int_{-\infty}^{\infty} g_{\ell'}^{(0)*}(\alpha';x)\cdot z_0\times f_\ell^{(0)}(\alpha;x)\,dx=\delta_{\ell\ell'}\delta(\alpha-\alpha') \quad (6)$$

where $\delta_{\ell\ell'}$ is Kronecker's delta, $\delta(\alpha-\alpha')$ Dirac's δ function and the symbol * signifies complex conjugate function. The longitudinal field components can be obtained from the transverse fields according to

$$E_Z^{(i)}=\frac{1}{j\omega\epsilon^{(i)}}\nabla\cdot\left(H_t^{(i)}\times z_0\right)$$

$$H_Z^{(i)}=\frac{1}{j\omega\mu_0}\nabla\cdot\left(z_0\times E_t^{(i)}\right) \quad (7)$$

where

$$\epsilon^{(i)}=\begin{cases}\epsilon_r\epsilon_0 & \text{(region (3))}\\ \epsilon_0 & \text{(other regions).}\end{cases}$$

Substituting (1), (3), and (7) into Maxwell's field equation and applying the orthonormal properties of the vector mode functions (6), we obtain the following transmission-line equations:

$$-\frac{dV_\ell^{(i)}}{dz}=ja_\ell^{(i)}I_\ell^{(i)}$$

$$-\frac{dI_\ell^{(i)}}{dz}=jc_\ell^{(i)}V_\ell^{(i)} \quad (8)$$

$a_\ell^{(i)}$ and $c_\ell^{(i)}$ are given by

$$a_1^{(i)}=\omega\mu_0-\frac{K^{(i)^2}}{\omega\epsilon^{(i)}} \qquad a_2^{(i)}=\omega\mu_0$$

$$c_1^{(i)}=\omega\epsilon^{(i)} \quad c_2^{(i)}=\omega\epsilon^{(i)}-\frac{K^{(i)^2}}{\omega\mu_0} \quad (9)$$

$$K^{(i)}=\begin{cases}K_n, & \text{(region (2))}\\ K, & \text{(other regions).}\end{cases}$$

The boundary conditions to be satisfied are expressed as follows:

1) $z=t;\ -W/2<x<W/2$

$$E_t^{(1)}=E_t^{(2)}=\bar{E}_a \quad (10a)$$

$$H_t^{(1)}=H_t^{(2)} \quad (10b)$$

2) $z=0;\ -W/2<x<W/2$

$$E_t^{(2)}=E_t^{(3)}=\bar{E}_b \quad (11a)$$

$$H_t^{(2)}=H_t^{(3)} \quad (11b)$$

3) $z=-h;\ -\infty<x<\infty$

$$E_t^{(3)}=E_t^{(4)} \quad (12a)$$

$$H_t^{(3)}=H_t^{(4)} \quad (12b)$$

where $\bar{E}_a$ and $\bar{E}_b$ are the transverse electric fields at the slot surfaces $z=t$ and $z=0$, respectively.

Applying the continuity conditions (10a), (11a), and (12) to the general solutions of the differential equations (8), the mode voltages and currents in each region are expressed as

$$V_\ell^{(1)}(\alpha;z)=T_\ell^{(1)}(\alpha;z|t)\bar{V}_{\ell a}(\alpha)$$

$$I_\ell^{(1)}(\alpha;z)=Y_\ell^{(1)}(\alpha;z|t)\bar{V}_{\ell a}(\alpha)$$

$$V_\ell^{(2)}(\alpha_n;z)=T_\ell^{(2)}(\alpha_n;z|t)\bar{V}_{\ell a}(\alpha_n)+T_\ell^{(2)}(\alpha_n;z|0)\bar{V}_{\ell b}(\alpha_n)$$

$$I_\ell^{(2)}(\alpha_n;z)=Y_\ell^{(2)}(\alpha_n;z|t)\bar{V}_{\ell a}(\alpha_n)+Y_\ell^{(2)}(\alpha_n;z|0)\bar{V}_{\ell b}(\alpha_n)$$

$$V_\ell^{(3)}(\alpha;z)=T_\ell^{(3)}(\alpha;z|0)\bar{V}_{\ell b}(\alpha)$$

$$I_\ell^{(3)}(\alpha;z)=Y_\ell^{(3)}(\alpha;z|0)\bar{V}_{\ell b}(\alpha) \quad (13)$$

where the Green's functions $T_\ell^{(i)}$ and $Y_\ell^{(i)}$ are given in the Appendix. The mode voltages at the slot surfaces $\bar{V}_{\ell a}$ and $\bar{V}_{\ell b}$ are expressed in terms of the transverse electric fields at the slot surfaces $\bar{E}_a$ and $\bar{E}_b$:

$$\bar{V}_{\ell}{}_b^a(\alpha)=\int_{-\infty}^{\infty} g_\ell^{(0)*}(\alpha;x')\cdot z_0\times\bar{E}{}_b^a(x',y')e^{j\beta_0 y'}\,dx' \quad (14)$$

$$\bar{V}_{\ell}{}_b^a(\alpha_n)=\int_{-\infty}^{\infty} g_\ell^{(c)*}(\alpha_n;x')\cdot z_0\times\bar{E}{}_b^a(x',y')e^{j\beta_0 y'}\,dx'.$$

The electromagnetic fields in each region can be obtained by substituting (13) into (1) and (3). The application of the remaining boundary conditions (10b) and (11b) leads us to the integral equations on the electric fields at the slot surfaces $\bar{E}_a$ and $\bar{E}_b$, and the unknown propagation constant β_0. $\bar{E}_a$ and $\bar{E}_b$ may be expressed in terms of β_0 as

$$\bar{E}{}_b^a(x',y')=\left\{x_0 e_{x}{}_b^a(x')+y_0 e_{y}{}_b^a(x')\right\}e^{-j\beta_0 y'}. \quad (15)$$

Thus the integral equations are expressed as follows:

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{K^2}\left[\left(b_1^{(1)}-b_2^{(1)}\right)\alpha\beta_0\tilde{e}_{xa}(\alpha)+\left(b_1^{(1)}\beta_0^2+b_2^{(1)}\alpha^2\right)\tilde{e}_{ya}(\alpha)\right]e^{-j\alpha x}\,d\alpha$$
$$=\frac{2}{W}\sum_{n=0}^{\infty}\frac{\sin(\alpha_n x)}{K_n^2}\left[j\left(b_1^{(2)}-b_2^{(2)}\right)\alpha_n\beta_0\left\{\coth(\kappa_n t)\tilde{e}_{xa}(\alpha_n)-\operatorname{cosech}(\kappa_n t)\tilde{e}_{xb}(\alpha_n)\right\}+\left(b_1^{(2)}\beta_0^2+b_2^{(2)}\alpha_n^2\right)\cdot\left\{-\coth(\kappa_n t)\tilde{e}_{ya}(\alpha_n)+\coth(\kappa_n t)\tilde{e}_{yb}(\alpha_n)\right\}\right] \quad (16a)$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{K^2}\left[\left(b_1^{(1)}\alpha^2+b_2^{(1)}\beta_0^2\right)\tilde{e}_{xa}(\alpha)+\left(b_1^{(1)}-b_2^{(1)}\right)\alpha\beta_0\tilde{e}_{ya}(\alpha)\right]e^{-j\alpha x}\,d\alpha$$
$$=\frac{2}{W}\sum_{n=0}^{\infty}\frac{\cos(\alpha_n\kappa)}{K_n^2}\left[\left(b_1^{(2)}\alpha_n^2+b_2^{(2)}\beta_0^2\right)\left\{-\coth(\kappa_n t)\tilde{e}_{xa}(\alpha_n)+\operatorname{cosech}(\kappa_n t)\tilde{e}_{xb}(\alpha_n)\right\}-j\left(b_1^{(2)}-b_2^{(2)}\right)\cdot\alpha_n\beta_0\left\{\coth(\kappa_n t)\tilde{e}_{ya}(\alpha_n)-\operatorname{cosech}(\kappa_n t)\tilde{e}_{yb}(\alpha_n)\right\}\right] \quad (16b)$$

$$\frac{2}{W}\sum_{n=0}^{\infty}\frac{\sin(\alpha_n x)}{K_n^2}\left[j\left(b_1^{(2)}-b_2^{(2)}\right)\alpha_n\beta_0\left\{-\operatorname{cosech}(\kappa_n t)\tilde{e}_{xa}(\alpha_n)+\coth(\kappa_n t)\tilde{e}_{xb}(\alpha_n)\right\}+\left(b_1^{(2)}\beta_0^2+b_2^{(2)}\alpha_n^2\right)\cdot\left\{\operatorname{cosech}(\kappa_n t)\tilde{e}_{ya}(\alpha_n)-\coth(\kappa_n t)\tilde{e}_{yb}(\alpha_n)\right\}\right]$$
$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{K^2}\left[\left(b_1^{(3)}-b_2^{(3)}\right)\alpha\beta_0\tilde{e}_{xb}(\alpha)+\left(b_1^{(3)}\beta_0^2+b_2^{(3)}\alpha^2\right)\tilde{e}_{yb}(\alpha)\right]e^{-j\alpha x}\,d\alpha \quad (16c)$$

$$\frac{2}{W}\sum_{n=0}^{\infty}\frac{\cos(\alpha_n x)}{K_n^2}\left[(b_1^{(2)}\alpha_n^2+b_2^{(2)}\beta_0^2)\{-\operatorname{cosech}(\kappa_n t)\tilde{e}_{xa}(\alpha_n)+\coth(\kappa_n t)\tilde{e}_{xb}(\alpha_n)\}-j(b_1^{(2)}-b_2^{(2)})\cdot\alpha_n\beta_0\{\operatorname{cosech}(\kappa_n t)\tilde{e}_{ya}(\alpha_n)-\coth(\kappa_n t)\tilde{e}_{yb}(\alpha_n)\}\right]$$
$$=\frac{-1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{K^2}\left[(b_1^{(3)}\alpha^2+b_2^{(3)}\beta_0^2)\tilde{e}_{xb}(\alpha)+(b_1^{(3)}-b_2^{(3)})\alpha\beta_0\tilde{e}_{yb}(\alpha)\right]e^{-j\alpha x}d\alpha \quad (16d)$$

where

$$b_1^{(1)}=\frac{\omega\epsilon_0}{\kappa_0}\quad b_2^{(1)}=-\frac{\kappa_0}{\omega\mu_0}$$
$$b_1^{(2)}=\frac{\omega\epsilon_0}{\kappa_n}\quad b_2^{(2)}=-\frac{\kappa_n}{\omega\mu_0}\epsilon^2(n)$$

$$b_1^{(3)}=b_1^{(1)}\frac{1+\epsilon_r\frac{\kappa_0}{\kappa}\tan(Kh)}{1-\frac{\kappa}{\epsilon_r\kappa_0}\tan(Kh)}\quad b_2^{(3)}=b_2^{(1)}\frac{1-\frac{\kappa}{\kappa_0}\tan(\kappa h)}{1+\frac{\kappa_0}{\kappa}\tan(\kappa h)}$$

$$\kappa_0=\sqrt{K^2-\omega^2\epsilon_0\mu_0}\quad \kappa_n=\sqrt{K_n^2-\omega^2\epsilon_0\mu_0}$$
$$\kappa=\sqrt{\omega^2\epsilon\mu_0-K^2} \quad (17)$$

and

$$\tilde{e}_{\substack{x\\y}i}(\alpha)=\int_{-W/2}^{W/2}e_{\substack{x\\y}i}(x')e^{j\alpha x'}dx'$$
$$\tilde{e}_{xi}(\alpha_n)=\int_{-W/2}^{W/2}e_{xi}(x')\cos(\alpha_n x')dx'$$
$$\tilde{e}_{yi}(\alpha_n)=\int_{-W/2}^{W/2}e_{yi}(x')\sin(\alpha_n x')dx',\qquad i=a,b \quad (18)$$

where x lies within the slot region $|x|\leqslant W/2$.

III. Method of Solution

In this section, the determinantal equation for the propagation constant β_0 will be derived by applying Galerkin's method to the integral equations (16).

As a first step we expand the unknown slot fields e_{xi} and $e_{yi} (i=a,b)$ in terms of known basis functions f_{xk} and f_{yk} as follows:

$$e_{xa}(x)=\sum_{k=1}^{N}a_{xk}f_{xk}(x)$$
$$e_{ya}(x)=j\sum_{k=1}^{N}a_{yk}f_{yk}(x)$$
$$e_{xb}(x)=\sum_{k=1}^{N}b_{xk}f_{xk}(x)$$
$$e_{yb}(x)=j\sum_{k=1}^{N}b_{yk}f_{yk}(x) \quad (19)$$

where $a_{\substack{x\\y}k}$ and $b_{\substack{x\\y}k}$ are unknown coefficients. After substituting (19) into (16), we multiply (16a) and (16c) by $f_{ym}(x)$, and (16b) and (16d) by $f_{xm}(x)$, and then integrate them with respect to x. The resultant equations are written as

$$\frac{1}{2\pi}\sum_{k=1}^{N}\int_{-\infty}^{\infty}d\alpha\frac{1}{K^2}\tilde{f}_{ym}^{*}(\alpha)\left[-jP^{(1)}(\alpha)\tilde{f}_{xk}(\alpha)a_{xk}+Q^{(1)}(\alpha)\tilde{f}_{yk}(\alpha)a_{yk}\right]$$
$$=\frac{2}{W}\sum_{k=1}^{N}\sum_{n=1}^{\infty}\frac{1}{K_n^2}\tilde{f}_{ym}(\alpha_n)\left[P^{(2)}(\alpha_n)\tilde{f}_{xk}(\alpha_n)\{\coth(\kappa_n t)a_{xk}-\operatorname{cosech}(\kappa_n t)b_{xk}\}+Q^{(2)}(\alpha_n)\tilde{f}_{yk}(\alpha_n)\{-\coth(\kappa_n t)a_{yk}+\operatorname{cosech}(\kappa_n t)b_{yk}\}\right],\quad m=1,2,\cdots,N \quad (20a)$$

$$\frac{1}{2\pi}\sum_{k=1}^{N}\int_{-\infty}^{\infty}d\alpha\frac{1}{K^2}\tilde{f}_{xm}^{*}(\alpha)\left[R^{(1)}(\alpha)\tilde{f}_{xk}(\alpha)a_{xk}+jP^{(1)}(\alpha)\tilde{f}_{yk}(\alpha)a_{yk}\right]$$
$$=\frac{2}{W}\sum_{k=1}^{N}\sum_{n=1}^{\infty}\frac{1}{K_n^2}\tilde{f}_{xm}(\alpha_n)\left[R^{(2)}(\alpha_n)\tilde{f}_{xk}(\alpha_n)\cdot\{-\coth(\kappa_n t)a_{xk}+\operatorname{cosech}(\kappa_n t)b_{xk}\}+P^{(2)}(\alpha_n)\tilde{f}_{yk}(\alpha_n)\{\coth(\kappa_n t)a_{yk}-\operatorname{cosech}(\kappa_n t)b_{yk}\}\right],\quad m=1,2,\cdots,N \quad (20b)$$

$$\frac{2}{W}\sum_{k=1}^{N}\sum_{n=0}^{\infty}\frac{1}{K_n^2}\tilde{f}_{ym}(\alpha_n)\left[P^{(2)}(\alpha_n)\tilde{f}_{xk}(\alpha_n)\cdot\{-\operatorname{cosech}(\kappa_n t)a_{xk}+\coth(\kappa_n t)b_{xk}\}+Q^{(2)}(\alpha_n)\tilde{f}_{yk}(\alpha_n)\cdot\{\operatorname{cosech}(\kappa_n t)a_{yk}-\coth(\kappa_n t)b_{yk}\}\right]$$
$$=\frac{1}{2\pi}\sum_{k=1}^{N}\int_{-\infty}^{\infty}d\alpha\frac{1}{K^2}\tilde{f}_{ym}^{*}(\alpha)\left[-jP^{(3)}(\alpha)\tilde{f}_{xk}(\alpha)b_{xk}+Q^{(3)}(\alpha)\tilde{f}_{yk}(\alpha)b_{yk}\right],\quad m=1,2,\cdots,N \quad (20c)$$

$$\frac{2}{W}\sum_{k=1}^{N}\sum_{n=0}^{\infty}\frac{1}{K_n^2}\tilde{f}_{xm}(\alpha_n)\left[R^{(2)}(\alpha_n)\tilde{f}_{xk}(\alpha_n)\cdot\{-\operatorname{cosech}(\kappa_n t)a_{xk}+\coth(\kappa_n t)b_{xk}\}+P^{(2)}(\alpha_n)\tilde{f}_{yk}(\alpha_n)\cdot\{\operatorname{cosech}(\kappa_n t)a_{yk}-\coth(\kappa_n t)b_{yk}\}\right]$$
$$=\frac{-1}{2\pi}\sum_{k=1}^{N}\int_{-\infty}^{\infty}d\alpha\frac{1}{K^2}\tilde{f}_{xm}^{*}(\alpha)\left[R^{(3)}(\alpha)\tilde{f}_{xk}(\alpha)b_{xk}+jP^{(3)}(\alpha)\tilde{f}_{yk}(\alpha)b_{yk}\right],\quad m=1,2,\cdots,N \quad (20d)$$

where

$$P^{(i)}=(b_1^{(i)}-b_2^{(i)})\alpha^{(i)}\beta_0$$
$$Q^{(i)}=b_1^{(i)}\beta_0^2+b_2^{(i)}\alpha^{(i)^2}$$
$$R^{(i)}=b_1^{(i)}\alpha^{(i)^2}+b_2^{(i)}\beta_0^2,\qquad \alpha^{(i)}=\begin{cases}\alpha_n, & \text{(region (2))}\\ \alpha, & \text{(other regions)}\end{cases} \quad (21)$$

and

$$\tilde{f}_{\substack{x\\y}k}(\alpha)=\int_{-W/2}^{W/2} f_{\substack{x\\y}k}(x)e^{j\alpha x}\,dx$$

$$\tilde{f}_{xk}(\alpha_n)=\int_{-W/2}^{W/2} f_{xk}(x)\cos(\alpha_n x)\,dx$$

$$\tilde{f}_{yk}(\alpha_n)=\int_{-W/2}^{W/2} f_{yk}(x)\sin(\alpha_n x)\,dx. \tag{22}$$

This set of simultaneous equations on the unknown $a_{\substack{x\\y}k}$ and $b_{\substack{x\\y}k}$ is homogeneous, and may be written in the following matrix form:

$$[G(\beta_0)]\begin{bmatrix}\bar{a}_x\\ \bar{a}_y\\ \bar{b}_x\\ \bar{b}_y\end{bmatrix}=0 \tag{23}$$

where $[G(\beta_0)]$ is a square matrix of order $4N$, and $\bar{a}_x$, $\bar{a}_y, \bar{b}_x$, and $\bar{b}_y$ are column matrices of the unknowns a_{xk}, a_{yk}, b_{xk}, and b_{yk}, respectively.

For (23) to yield nontrivial solutions, the determinant of the coefficient matrix $[G(\beta_0)]$ must be zero. This condition results in the determinantal equation for the propagation constant β_0

$$\det[G(\beta_0)]=0. \tag{24}$$

It remains only to select the basis functions $f_{xk}(x)$ and $f_{yk}(x)$. It is desirable that the edge effect of slot fields should be accounted for, and that the approximation to the slot fields should be systematically improved by increasing the number of basis functions. In view of these requirements, the following families of functions are adopted for basis functions:

$$f_{xk}(x)=\frac{T_{k-1}\left(\frac{2x}{W}\right)}{\sqrt{1-\left(\frac{2x}{W}\right)^2}}$$

$$f_{yk}(x)=U_k\left(\frac{2x}{W}\right) \tag{25}$$

where $T_k(y)$ are Chebyshev's polynomials of the first kind and $U_k(y)$ are Chebyshev's polynomials of the second kind. The forms of these basis functions are shown in Fig. 2.

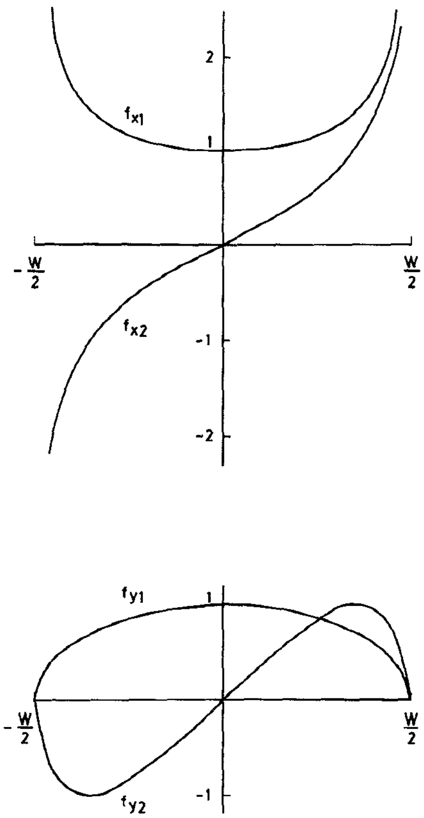

Fig. 2. The functional forms of basis functions.

IV. Numerical Computations

The solutions of the determinantal equation (24) were obtained using a digital computer. The integrals and summations in matrix $[G]$ can be evaluated with high accuracy, because they converge extremely rapidly.

To investigate the computation accuracy of the method, the propagation constants are computed assuming that the thickness of the metal coating t equals to zero. In such a case $e_{\substack{x\\y}a}=e_{\substack{x\\y}b}$, hence $a_{\substack{x\\y}k}=b_{\substack{x\\y}k}$ and $[G]$ is a $2N\times 2N$ matrix. In Table I, the computed results, using the different order of approximation, are shown and compared with those by other method [1]. The rapid convergence is obtained, which is the result of the adequate choice of the basis functions (25).

In this arrangement, the coefficients of the basis functions $a_{\substack{x\\y}k}$ were also calculated for $N=4$. For the dominant mode, $|a_{y2}/a_{x1}|=7.1\times10^{-3}$, $|a_{x3}/a_{x1}|=3.0\times10^{-2}$, $|a_{y4}/a_{x1}|=1.3\times10^{-5}$, and $a_{y1}=a_{x2}=a_{y3}=a_{x4}=0$. For the first higher order mode, on the contrary, $|a_{x2}/a_{y1}|=2.6$, $|a_{y3}/a_{y1}|=8.6\times10^{-3}$, $|a_{x4}/a_{y1}|=2.5\times10^{-3}$, and $a_{x1}=a_{y2}=a_{x3}=a_{y4}=0$. These results as well as those in Table I show that retaining only one term ($N=1$) is sufficient for the dominant mode, while two terms ($N=2$) are necessary for the first higher order mode.

Fig. 3 shows the dispersion characteristics of slot line, where the normalized propagation constants for the dominant and the first higher order mode are reported. In the computations, the first two basis functions ($N=2$) are used for both the dominant and first higher order mode. The numerical data are compared with the results of Cohn [1], and the agreement is quite good.

Figs. 4 and 5 show the effect of the metal-coating thickness on the propagation constant and the characteristic impedance, respectively. The definition for characteristic impedance is not uniquely specified due to the propagation of the hybrid mode. The definition chosen here is

$$Z_0=\frac{V_0^2}{2P_{\text{ave}}} \tag{26}$$

TABLE I
PROPAGATION CONSTANT $\beta_0/\omega\sqrt{\epsilon_0\mu_0}$ (N = THE NUMBER OF BASIS FUNCTIONS)

N	Dominant mode	First higher-order mode
1	2.2777	-
2	2.2777	1.6829
3	2.2776	1.6829
4	2.2776	1.6829
Cohn's [1]	2.284	-

ε_r = 9.6, h = 1 (mm), W = 1 (mm), f = 25 (GH_z)

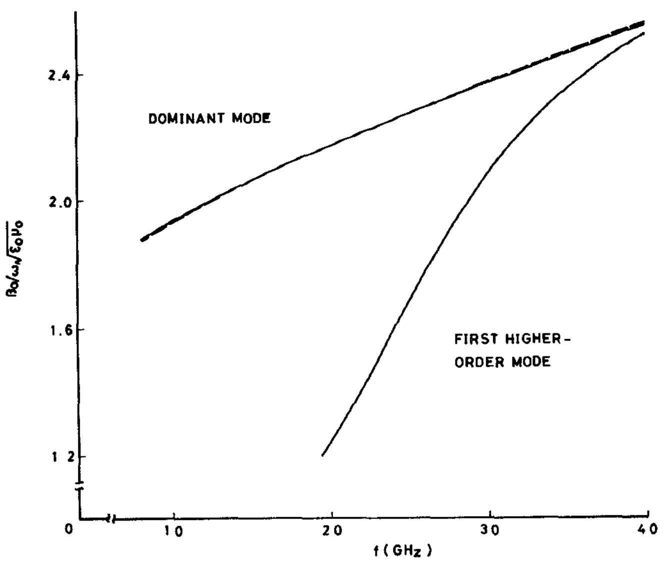

Fig. 3. Normalized propagation constant for the dominant and first higher order mode in slot line: $\epsilon_r=9.6$, $h=1$ (mm), $W=1$ (mm), $t=0$. Solid line represents present method and broken line represents Cohn's method [1].

where V_0 is the peak voltage defined by

$$V_0=\frac{1}{2}\left\{\int_{-W/2}^{W/2}\bar{E}_a\cdot x_0\,dx+\int_{-W/2}^{W/2}\bar{E}_b\cdot x_0\,dx\right\} \tag{27}$$

and P_{ave} is the average power flow along the y direction. The first term of the basis functions is retained in these computations ($N=1$), therefore results for only the dominant mode are presented. In this case $|\mathcal{Y}1/a_{x1}|=0$, therefore, the determinantal equation (24) becomes identical with in [3, eq. 14] and Fig. 4 here is the same as in [3, fig. 2]. It is noted that the effect of the thickness of the metal coating is the decrease in the propagation constant and the characteristic impedance.

The typical computation time is about 16 s for the zero thickness ($N=2$) and about 10 s for the finite-thickness

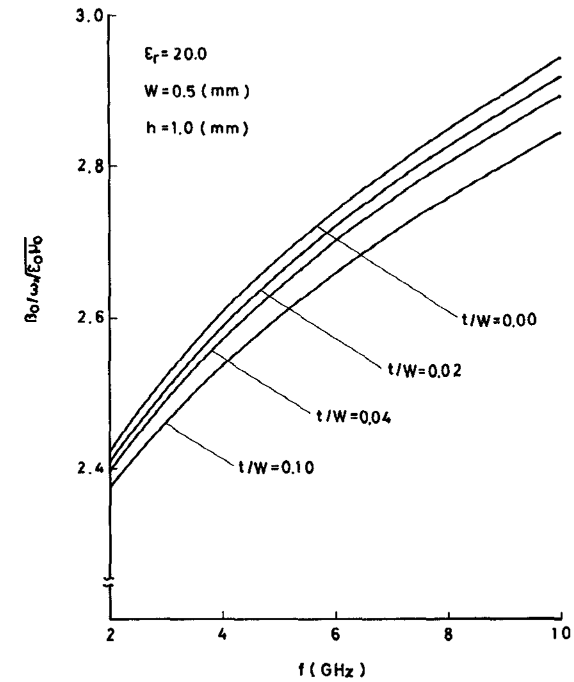

Fig. 4. Normalized propagation constant.

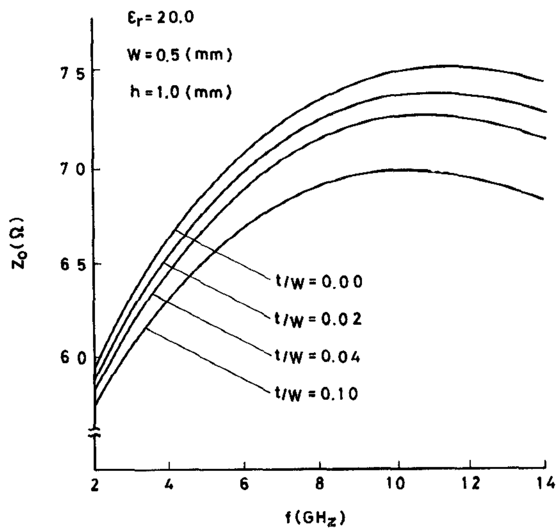

Fig. 5. Characteristic impedance.

($N=1$) using the electronic computer FACOM 230-75.

V. CONCLUSIONS

This paper described a hybrid mode analysis of the slot line. The formulation uses the network analytical methods of electromagnetic fields and numerical procedure is based on Galerkin's method. The propagation constants for the dominant and first higher order mode as well as the effect of the metal-coating thickness on the propagation constant and the characteristic impedance are computed. It is found that the rapid convergence is obtained because of the application of Galerkin's method and the adequate choice of the basis functions. Numerical results obtained for the dominant mode have been compared with other available data and are in good agreement. This method itself is general and applicable to a number of other structures, e.g., microstrip and coplanar waveguide. Further results will be reported in near future.

Appendix

Green's functions $T_\ell^{(i)}$ and $Y_\ell^{(i)}$ are given by

$$T_\ell^{(1)}(\alpha; z|t) = e^{-\kappa_0(z-t)}$$

$$Y_\ell^{(1)}(\alpha; z|t) = y_\ell^{(1)} e^{-\kappa_0(z-t)}$$

$$T_\ell^{(2)}(\alpha_n; z|z') = \frac{\sinh \kappa_n |z-z'|}{\sinh \kappa_n t}$$

$$Y_\ell^{(2)}(\alpha_n; z|z') = \pm y_\ell^{(2)} \frac{\cosh \kappa_n (z-z')}{\sinh \kappa_n t}, \qquad z \gtrless z'$$

$$T_\ell^{(3)}(\alpha; z|0) = \frac{\cos \kappa(z+h) + j\dfrac{y_\ell^{(1)}}{y_\ell^{(3)}} \sin \kappa(z+h)}{\cos \kappa h + j\dfrac{y_\ell^{(1)}}{y_\ell^{(3)}} \sin \kappa h}$$

$$Y_\ell^{(3)}(\alpha; z|0) = -y_\ell^{(1)} \frac{\cos \kappa(z+h) + j\dfrac{y_\ell^{(3)}}{y_\ell^{(1)}} \sin \kappa(z+h)}{\cos \kappa h + j\dfrac{y_\ell^{(1)}}{y_\ell^{(3)}} \sin \kappa h}$$

where

$$\kappa_0 = \sqrt{K^2 - \omega^2 \epsilon_0 \mu_0} \qquad \kappa_n = \sqrt{K_n^2 - \omega^2 \epsilon_0 \mu_0}$$

$$\kappa = \sqrt{\omega^2 \epsilon \mu_0 - k^2} \qquad \epsilon = \epsilon_r \epsilon_0$$

$$y_1^{(1)} = j\frac{\omega \epsilon_0}{\kappa_0} \qquad y_2^{(1)} = -j\frac{\kappa_0}{\omega \mu_0}$$

$$y_1^{(2)} = j\frac{\omega \epsilon_0}{\kappa_n} \qquad y_2^{(2)} = -j\frac{\kappa_n}{\omega \mu_0}$$

$$y_1^{(3)} = \frac{\omega \epsilon}{\kappa} \qquad y_2^{(3)} = \frac{\kappa}{\omega \mu_0}.$$

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